

ПІДГОТОВКА СПЕЦІАЛІСТІВ В ОБЛАСТІ РАДІОТЕХНІКИ ТА ТЕЛЕКОМУНІКАЦІЙ

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POSSIBILITIES OF USING CMS MAPLE TO STUDY RANDOM VARIABLE DISTRIBUTION LAWS

Introduction

One of the fundamental notion of probability theory is the notion of a random variable. If we know all possible values which the random variable was assumed and probability for every their variable then we find it theoretically defined distribution. Thus distribution law of a random variable defined its probability as function defined on event set. Random variables, distribution laws and other performances are used during applied and fundamental experiment in many science and technique areas. For this reason, the study of random variables distribution laws is actual.

Statistical calculations without computer are difficult and require many functional and quintiles tables of standard laws. It does not contribute to feel the element of novelty in the material students are studying, to change problem conditions and so on. It requires much time to solve applied tasks and is inappropriate.

Today in the world market there are more than 1000 widest packages solved statistical data analysis problems at differential computer operational systems. Statistical applied programs divided on universal, semiuniversal, special packages and statistical expert systems. Specialized mathematical packages (SAS, SPSS, STATISTIKA, STATGRAPHICS) are not relevant for study. Their use for studying requires very high education level in mathematical statistics.

Thus to determine and research random variables distribution laws both in practical applications and in studying we must use special mathematical packages. The most extended of them are Mathcad, MatLab, Mathematica, Maple.

Most of the existing math packages allow users to operate at random variables, including the Computer Mathematics System (CMS) Maple [1].

Thus this article purpose is a description of the studying possibilities of the random variables distribution laws with CMS Maple and the application of the acquired skills in the independent work of students.

Choice of Computer Mathematics System

Computer Mathematics Systems are effective method to mathematic learning by students from Europe, USA, Japan, China and other countries. Unfortunately in our education system both students and teachers are used modern computer mathematics systems not enough. It very slowed down solving some problems to implement our education system in world education one where computer mathematics systems are in active using.

Publication analysis evident about intensive investigation on the implementation of information and communication technologies in particular computer oriented education systems.

Choice of Computer Mathematics System is depend on the ultimate goal of program using, tasks class, its appointment.

Didactical functions of such systems are:

- visual means data presenting (electron reference book with hypertext help system and intuitive interface, animate examples audio and video accompaniment);
- solving practical problems means, complex models research, thorough analysis of solving tasks' variants, development of practical skills of mathematical reasoning.

CMS can be divided into seven classes: systems for numerical calculations; table processors; matrix systems; systems for statistical calculations; systems for special calculations; systems for analytical calculations (computer algebra); universal systems [2].

CMS comparison was shown in table [3].

CMS comparison

CMS	Advantages	Disadvantages
<i>Mathematika</i>	Compatibility with computer platforms. 3D-graphics. Documents (notebook). Sound synthesis support.	Excessive copy protection. Targeting experienced users.
<i>Matlab</i>	Unique matrix tools, descriptor graphics, high speed calculations, adaptation to user's tasks and number of system packages.	Limited opportunities of symbolic calculations. High cost of the system and their packages.
<i>MathCad</i>	Quality graphics and visualization during calculations. Comfortable interface. Mathematical signs palette availability. Huge choice of electron books and libraries, operators and functions.	Symbolic mathematic limitation. Primitive programming. Cost of electron books and libraries.
<i>Maple</i>	Thoughtful kernel of symbolic calculations. Documents (notebook). High quality graphics. Convenient help system.	No sound synthesis.

Compared to other mathematical software packages, CMS Maple has several advantages that are of particular importance when solving statistical applied problems in the field of metrology, namely: possibility of symbolic calculations, operation of numbers with arbitrary accuracy, representation of graphs in dynamic mode, etc [1].

Statistics with Maple

As well known, random variables are divided into discrete and continuous ones. Accordingly, the random variables distribution laws are divided into distribution laws for discrete random variables and the distribution laws for continuous random variables.

Statistics in CMS Maple has a sophisticated packages suite to handle applications of various types and purposes. Maple commands for statistics work are intended for those categories of users who need an environment that makes it easy to move from one mathematical specialization to another without spending too much time transforming data and mastering a software variety.

The Maple Statistics Library has a large set of commands for analyzing data, computing various numerical characteristics of random variables, graphing their distribution laws, and for statistical data processing [4 – 8].

This library provides the ability to operate with the following laws for the discrete random variables distributions: Bernoulli, binomial, discrete uniform, empirical, geometric, hypergeometric, negative binomial (Pascal), Poisson; and continuous random variables: beta, Cauchy, χ^2 , Erlang, error (exponential power), exponential, Fisher, gamma, Gumbel, inverse gaussian (Wald), Laplace, logistic, log normal, Maxwell, Moyal, noncentral beta, noncentral χ^2 , noncentral f-distribution, noncentral t-distribution, normal (gaussian), Pareto, power, Rayleigh, Student-t, triangular, uniform (rectangular), von Mises, Weibull. In addition, it is possible to create new distribution laws and to study their properties.

The presence of such large list of random variables distribution laws allows them to be studied and investigated by students both in the course of laboratory or practical work, as well as during independent work, as well as in solving applied and theoretical problems in all fields of science and technology.

Below we showed the algorithm of the random variables distribution laws study. This algorithm consists of such stages:

1. To define the random variables distribution law.

2. To obtain the Cumulative Distribution Function expression.
3. To obtain the Probability Density Function expression.
4. To obtain the descriptive statistics expression.
5. To generate random sample according to the chosen distribution law.
6. To compare theoretical descriptive statistics' values and generated descriptive statistics' values.
7. To plot Probability Density Function graphic.

Let's take an example results which obtained using above mentioned algorithm for three distribution laws: one for discrete random variables, one for continuous random variables and one for created law.

Below are the results obtained using above mentioned algorithm to Poisson distribution law (discrete random variables), Maxwell distribution law (continuous random variables) and created distribution law.

Probability density function of the created distribution law was defined as

$$f(x) = \begin{cases} 0 & x < -1 \\ (x+1)^2 & -1 \leq x < 0 \\ 1-x^2 & 0 \leq x < 1 \\ 0 & x \geq 1 \end{cases} \quad (1)$$

This function was shown in fig. 1.

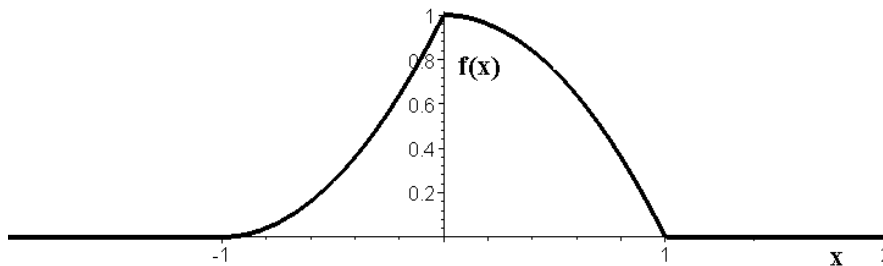


Fig. 1. Probability density function of the created distribution law

Descriptive statistics using classical approach were defined as:

Cumulative Distribution Function

$$F(x) = \int f(x)dx = \begin{cases} 0 & x < -1 \\ \frac{(x+1)^3}{3} & -1 \leq x < 0 \\ x - \frac{1}{3}x^3 + \frac{1}{3} & 0 \leq x < 1 \\ 1 & x \geq 1 \end{cases} \quad (2)$$

Expected value

$$M = \int_{-\infty}^{\infty} xf(x)dx = \frac{1}{6} \quad (3)$$

Variance

$$D = \int_{-\infty}^{\infty} (x-M)^2 f(x)dx = \frac{5}{36} \quad (4)$$

Standard Deviation

$$\sigma = \sqrt{D} = \frac{\sqrt{5}}{6} \quad (5)$$

Skewness

$$As = \frac{\int_{-\infty}^{\infty} (x - M)^3 f(x) dx}{\sigma^{3/2}} = -\frac{8\sqrt{5}}{125}. \quad (6)$$

Kurtosis

$$Ex = \frac{\int_{-\infty}^{\infty} (x - M)^4 f(x) dx}{D^2} = \frac{309}{125}. \quad (7)$$

To create a new distribution law we must use command **Distribution**.

First, define new distribution law as

f := simplify(piecewise(x < -1, 0, x < 0, (x+1)^2, x < 1, 1-x^2, x > 1, 0)):

Second, create this law

MyLaw := Distribution(PDF = unapply(f, x)):

Define random variables distribution laws by commands

> xx := Poisson(lambda); X := RandomVariables(xx):

> xx := Maxwell(alpha); Y := RandomVariables(xx):

> xx := MyLaw; Z := RandomVariables(xx):

For selected distribution laws we obtained cumulative distribution functions using command

CDF.

> CDF(X,p); CDF(Y,x); CDF(Z,x);

$$-\frac{\Gamma(2 + \text{floor}(p))}{(\text{floor}(p) + 1)!} + 1 + \frac{(\text{floor}(p) + 1)\Gamma(\text{floor}(p) + 1, \lambda)}{(\text{floor}(p) + 1)!}$$

$$\begin{cases} 0 & x < 0 \\ \frac{1}{3} \frac{\sqrt{2}x^3 \text{hypergeom}\left(\left[\frac{3}{2}\right], \left[\frac{5}{2}\right], -\frac{x^2}{2\alpha^2}\right)}{\alpha^3} & \text{otherwise} \end{cases}$$

$$\begin{cases} 0 & x < -1 \\ \frac{(x+1)^3}{3} & x \leq 0 \\ x - \frac{1}{3}x^3 + \frac{1}{3} & x \leq 1 \\ 1 & \text{otherwise} \end{cases}$$

For selected distribution laws we obtained probability density functions using command **PDF**.

> PDF(X,p); PDF(Y,x); PDF(Z,x);

$$\sum_{k=0}^{\infty} \frac{\lambda^k e^{(-\lambda)} \text{Dirac}(p - k)}{k!}$$

$$\begin{cases} 0 & x < 0 \\ \frac{\sqrt{2} \sqrt{\frac{1}{\pi}} x^2 e^{\left(-\frac{x^2}{2\alpha^2}\right)}}{\alpha^3} & \text{otherwise} \end{cases}$$

$$f(x) = \begin{cases} 0 & x < -1 \\ (x+1)^2 & -1 \leq x < 0 \\ 1-x^2 & 0 \leq x < 1 \\ 0 & \text{otherwise} \end{cases}$$

Using command **Mean** from Statistics library we obtained theoretical value of expected value and its empirical value for 250 elements sample to mentioned distribution laws.

```
> Mean(X); Mean(Y); Mean(Z);
λ
 $\frac{2\sqrt{2}\alpha}{\sqrt{\pi}}$ 
 $\frac{1}{6}$ 
> A:=Sample(X,250);B:= Sample(Y,250);C:= Sample(Z,250);
> Mean(A); Mean(B); Mean(C);
14.848
3.27
0.1667
```

Using command **Variance** and **StandardDeviation** from Statistics library we obtained theoretical value of Variance, Standard Deviation and its empirical value for 250 elements sample to mentioned distribution laws.

```
> Variance(X); Variance(Y); Variance(Z);
λ
 $\frac{\alpha^2(3\pi - 8)}{\pi}$ 
 $\frac{5}{36}$ 
> StandardDeviation(X); StandardDeviation(Y); StandardDeviation(Z);
 $\sqrt{\lambda}$ 
 $\alpha\sqrt{\frac{3\pi - 8}{\pi}}$ 
 $\frac{\sqrt{5}}{6}$ 
> Variance(A); Variance(B); Variance(C);
13.2298
1.799
0.138889
> StandardDeviation(A); StandardDeviation(B); StandardDeviation(C);
3.637
1.341
0.3727
```

Using command **Skewness** from Statistics library we obtained theoretical value of Skewness and its empirical value for 250 elements sample to mentioned distribution laws.

```
> Skewness(X); Skewness(Y); Skewness(Z);
 $\frac{1}{\sqrt{\lambda}}$ 
```

$$-\frac{2\sqrt{2}(5\pi-16)}{\pi^{(3/2)}\left(\frac{3\pi-8}{\pi}\right)^{(3/2)}}$$

$$-\frac{8\sqrt{5}}{125}$$

> Skewness(A); Skewness(B); Skewness(C);

0.25

0.3733

-0.143

Using command **Kurtosis** from Statistics library we obtained theoretical value of Kurtosis and its empirical value for 250 elements sample to mentioned distribution laws.

> Kurtosis(X); Kurtosis(Y); Kurtosis(Z);

$$\frac{3\lambda+1}{\lambda}$$

$$\frac{16\pi-192+15\pi^2}{(3\pi-8)^2}$$

309

125

> Kurtosis(A); Kurtosis(B); Kurtosis(C);

2.685

2.9374

2.472

Using command **DensityPlot** from Statistics library we plotted graphics to mentioned distribution laws shown in fig. 2 (Poisson (a) and Maxwell (b) distribution laws).

> DensityPlot(X, colour=blue, thickness=4);

> DensityPlot(Y, colour=blue, thickness=4, range=0..10);

> DensityPlot(Z, colour=black, thickness=4);

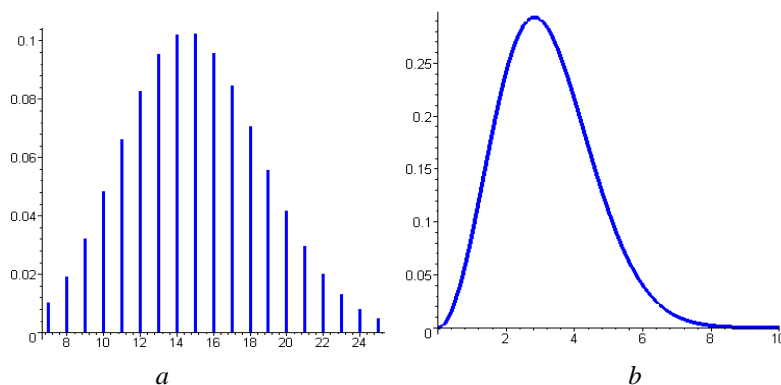


Fig. 2. Probability density function of the Poisson (a) and Maxwell (b) distribution laws

Conclusion

Thus, thanks to a powerful set of statistical tools, the ability to symbolically calculate and process expressions and data, the extensive CMS Maple, the extensive capabilities of graphically interpreting the results not only in static but also in dynamic form (two- and three-dimensional animation), it is advisable to use when studying the topic "Distribution Laws of random variables" in students' practical and independent work for further use of their acquired skills in solving applied science and technology problems.

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