

ELECTROMAGNETIC LATTICE «INVISIBILITY» OF THE RESONANCE CUBIC CRYSTAL MADE OF MAGNETODIELECTRIC SPHERES

Introduction

A problem of modeling a phenomenon of physical bodies “invisibility” at optic and roentgen wave bands is a serious direction of investigation in the applied electromagnetics. In this paper, properties of a limited crystal with a cubic lattice when a structural (lattice) resonance excited in it entails a phenomenon of electromagnetic lattice “invisibility” are analyzed. Considered here is the case equivalent to a roentgen optics of crystals when $a/\lambda' = 1$; $a/\lambda_g : 1$, $d, h, l/\lambda' : 1$; where a is the radius of the spheres; λ', λ_g lengths of the scattered wave outside and inside the spheres; d, h, l are the constants of the lattice. The problem solution is obtained on the basis of the second kind Fredholm integral equations of electromagnetics [1, 2, 3, 4].

Main part

Let us determine the scattered field from the known internal field of scatterers through electric $\overset{\cdot}{I}^e$ and magnetic $\overset{\cdot}{I}^m$ Hertz's potentials of the spatial lattice [1, 2, 3]

$$\begin{aligned}\overset{\cdot}{A}_{scat} &= (\nabla\nabla + k^2\varepsilon_0\mu_0)\overset{\cdot}{I}^e - ik\mu_0[\nabla, \overset{\cdot}{I}^m], \\ \overset{\cdot}{H}_{scat} &= (\nabla\nabla + k^2\varepsilon_0\mu_0)\overset{\cdot}{I}^m + ik\varepsilon_0[\nabla, \overset{\cdot}{I}^e].\end{aligned}\quad (1)$$

Having known internal field of the individual scatterers, present Hertz's potential $\overset{\cdot}{I}^e$ of the field scattered by the system of lattices as a superposition of Hertz's potentials of individual spheres of the lattices in the form

$$\overset{\cdot}{I}^e(\mathbf{r}, t) = \sum_{c=1}^C \left[\sum_p \sum_s \sum_t \frac{1}{k_1^3} (\sin k_1 a_c - k_1 a_c \cos k_1 a_c) \tilde{\delta} \left(\frac{\varepsilon_c - 1}{\varepsilon_0} \right) \overset{\cdot}{E}_{c(p,s,t)}^0(\mathbf{r}', t) \frac{e^{-ik_1 r_{c(p,s,t)}}}{r_{c(p,s,t)}} \right]_c, \quad (2)$$

$$r_{c(p,s,t)} = \sqrt{(x - x_{c,s})^2 + (y - y_{c,t})^2 + (z - z_{c,p})^2} \quad (3)$$

where coordinates (x, y, z) specify points of observation of the field outside the spheres; whereas $(x_{c,s}, y_{c,t}, z_{c,p})$ are coordinates of the points whereat centers of the scattering spheres of the lattice are located; $\overset{\cdot}{E}_{c(p,s,t)}^0(\mathbf{r}', t)$ is the internal field of the spheres to be found from the algebraic system of inhomogeneous equations of the quasi-stationary approximation [1]. Let us present one of the equations of this system for a simple lattice as

$$\begin{aligned}\overset{\cdot}{E}_{0c'(p',s',t')}(\mathbf{r}', t) &= \left(\frac{(\varepsilon_{c'ef} + 2\varepsilon_0) + \theta_{1c'}^2 \varepsilon_{c'ef} + i\theta_{1c'} (\varepsilon_{c'ef} + 2\varepsilon_0)}{3\varepsilon_0 e^{i\theta_{1c'} r}} \right) \overset{\cdot}{E}_{c'(p',s',t')}^0(\mathbf{r}', t) - \\ &- \sum_p \sum_s \sum_t \left\{ (\nabla\nabla + k^2\varepsilon_0\mu_0) \frac{1}{4\pi} \left(\frac{\varepsilon_{c'\delta} - 1}{\varepsilon_0} \right) W_{c'(p,s,t)}^e(\mathbf{r}) \overset{\cdot}{E}_{c'(p,s,t)}^0(\mathbf{r}', t) - \right. \\ &\left. c'(p, s, t) \neq c'(p', s', t') \right\}\end{aligned}$$

$$\begin{aligned}
& -ik\mu_0 \left[\nabla, \frac{1}{4\pi} \left(\frac{\mu_{cef}}{\mu_0} - 1 \right) W_{c'(p,s,t)}^m(\mathbf{r}) \overset{\mathbf{r}}{H}_{c'(p,s,t)}^0(\mathbf{r}', t) \right] \Bigg]_{c'} - \\
& - \sum_{\substack{c=1 \\ (c \neq c')}}^c \left(\sum_p \sum_s \sum_t \left\{ \left(\nabla \nabla + k^2 \varepsilon_0 \mu_0 \right) \frac{1}{4\pi} \left(\frac{\varepsilon_{cef}}{\varepsilon_0} - 1 \right) W_{c(p,s,t)}^e(\mathbf{r}) \overset{\mathbf{r}}{E}_{c(p,s,t)}^0(\mathbf{r}', t) - \right. \right. \\
& \left. \left. - ik\mu_0 \left[\nabla, \frac{1}{4\pi} \left(\frac{\mu_{c\dot{\gamma}\delta}}{\mu_0} - 1 \right) W_{c(p,s,t)}^m(\mathbf{r}) \overset{\mathbf{r}}{H}_{c(p,s,t)}^0(\mathbf{r}', t) \right] \right\} \right) \Bigg]_c,
\end{aligned}$$

where $W_{c(p,s,t)}^m(\mathbf{r}') = W_{c(p,s,t)}^e(\mathbf{r}') = \frac{4\pi}{k_1^3} (\sin k_1 a_{\bar{n}} - k_1 a_{\bar{n}} \cos k_1 a_{\bar{n}}) \frac{e^{-ik_1 r_{c'(p',s',t'),c(p,s,t)}}}{r_{c'(p',s',t'),c(p,s,t)}}$,

$$\varepsilon_{cef} = \varepsilon_c F(\theta), \quad \mu_{cef} = \mu_c F(\theta),$$

$$F(\theta) = 2(\sin \theta - \theta \cos \theta) / (\theta^2 - 1) \sin \theta + \theta \cos \theta;$$

$$\theta = ka_c \sqrt{\varepsilon_c \mu_c}.$$

The field scattered by the system of orthogonal lattices can be found from (1) taking into account (2) as

$$\begin{aligned}
\overset{\mathbf{r}}{E}_{scat}(\mathbf{r}, t) &= \sum_{c=1}^c \left[\sum_p \sum_s \sum_t \frac{1}{k_1^3} (\sin k_1 a_c - k_1 a_c \cos k_1 a_c) \times \right. \\
& \times \left\{ \left(\frac{\varepsilon_{cef}}{\varepsilon_0} - 1 \right) \overset{\mathbf{r}}{L}_c \overset{\mathbf{r}}{E}_{c(p,s,t)}^0(\mathbf{r}') - ik\mu_0 \left(\frac{\mu_{cef}}{\mu_0} - 1 \right) \times \right. \\
& \left. \left. \times \overset{\mathbf{r}}{P}_c \overset{\mathbf{r}}{H}_{c(p,s,t)}^0(\mathbf{r}') \right\} e^{i(\omega t - k_1 r_{c(p,s,t)})} \right]_c
\end{aligned} \tag{4}$$

where $\overset{\mathbf{r}}{L}_c$ and $\overset{\mathbf{r}}{P}_c$ are the functional matrices of scattering. Expression (4) describes propagating and damped constituents of the scattered field inside and outside the crystal in the Fresnel and Fraunhofer regions.

The full field at an arbitrary, which is located outside the spheres, point of medium is defined as

$$\begin{aligned}
\overset{\mathbf{r}}{E}(\mathbf{r}, t) &= \overset{\mathbf{r}}{E}_0(\mathbf{r}, t) + \overset{\mathbf{r}}{E}_{scat}(\mathbf{r}, t), \\
\overset{\mathbf{r}}{H}(\mathbf{r}, t) &= \overset{\mathbf{r}}{H}_0(\mathbf{r}, t) + \overset{\mathbf{r}}{H}_{scat}(\mathbf{r}, t)
\end{aligned} \tag{5}$$

where $\overset{\mathbf{r}}{E}_0(\mathbf{r}, t)$, $\overset{\mathbf{r}}{H}_0(\mathbf{r}, t)$ are the undisturbed fields of the scattered wave.

Power density of the scattered (1) and full (5) electromagnetic field can be found from the relation

$$\omega = \frac{1}{8\pi} (\overset{\mathbf{r}}{E}^2 + \overset{\mathbf{r}}{H}^2) \tag{6}$$

The numerical analysis of expressions (1) and (6) has been carried out for the resonance cube-shaped crystal. Its results are given in Figs 1, 2. The relationship between the resonance length of the scattered plane wave $\lambda_p^{\dot{\gamma}\delta}$ and constant d of the cubic lattice of the crystal is chosen in the form [3]

$$\lambda_p^{st} = 0,8d. \tag{7}$$

Under this condition there occur the structural (lattice) resonance with index $n = 3$ (Fig.1 a) and the associated with it phenomenon of electromagnetic lattice “invisibility” when the scattered wave does not experience reflection and mainly passes through the crystal (Fig. 2 c, d). Shapes of resonance curves here depend on the algebraic sum of the fields with corresponding phase multipliers (4).

The occurrence of regions with resonance propagation of the scattered wave is connected with this effect (Fig. 1 b, c).

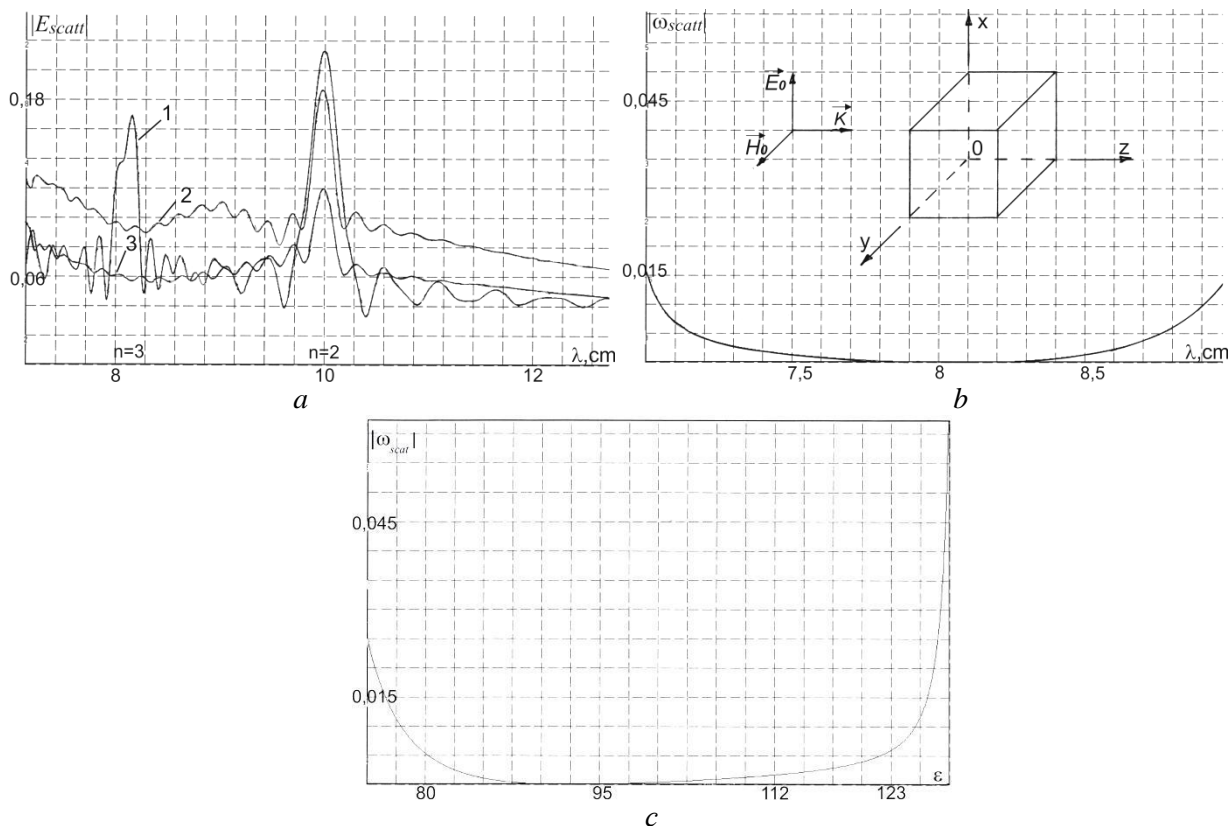


Fig.1. Dispersive dependences for $|A_{scatt}|$ and $|\omega_{scatt}|$ of the resonance cubic crystal

Position of $|\omega_{scat}|$ (6) at fixed distances on both crystal sides along z-axis (Fig.1 b, c) at Fresnel and Fraunhofer regions in the direction of scattered wave propagation allows estimating a widths of regions of resonance wave propagation.

The number of spheres in the crystal is $N = 64000$; the radius of the spheres $a = 0,5$ cm; the permittivity and permeability of the spheres $\epsilon = 95$ and $\mu = 1$ and those of their surroundings are $\epsilon_0 = \mu_0 = 1$; a constant of the cubic crystal lattice $d = h = l = 10$ cm.

Fig.1a shows the dispersive dependences $|E_{scat}|$ (4) at the cube center (curve 1) in the vicinity of the cube corner (curves 2, 3). Indices $n=2, 3, \dots$ specify the number of the structural resonance [3].

Shown in Fig. 1 b, c are the dispersive dependences (6) outside the cube for distances ± 10000 cm along a z-axis.

In Fig 2 a, b, c, and d, a structure of the internal field of the resonance cube in the directions of x, y, z-axes for $|\omega_{full}|$ and in the direction of z-axis for $|\omega_{scat}|$ (6) is considered.

Conclusion

Using structural (lattice) resonances of the crystals, whose occurrence is related with the presence of certainly shaped external surface enveloping the crystal, it is possible to create conditions for occurrence of the resonance lattice “invisibility” for electromagnetic waves scattering by the crystals and to form a structure of the internal field of the crystal.

The presented in the paper results of our investigations may be used when creating devices in which stealth technologies are employed.

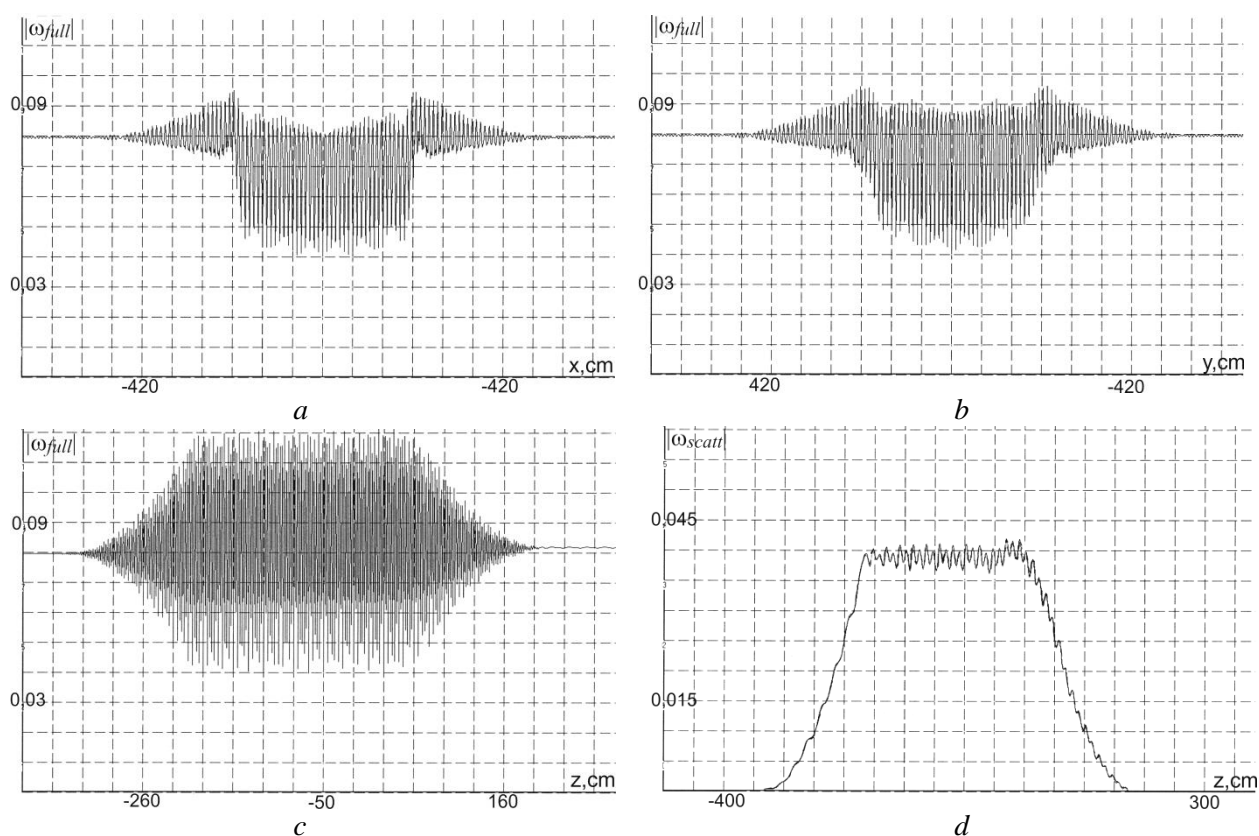


Fig.2. Dependences for $|\omega_{full}|$ and $|\omega_{scat}|$ of the internal field of the resonance cubic crystal

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